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Guaranteed Performance Control of Nonlinear Systems with Application to Flexible Space Structure

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This paper considers the problem of controlling nonlinear uncertain systems arising from flexible aerospace structures. We begin by addressing the tracking problem for a class of nonlinear dynamic systems with modeling uncertainties and external disturbances. New control algorithms that accommodate modeling uncertainties are proposed. It is shown that these algorithms not only guarantee system stability but also achieve certain bounded performance index and are readily applicable in vibration suppression of large flexible space structures. Numerical verification of the proposed strategy is demonstrated via the lumped mass model of a hypersonic aerospace flight vehicle structure.

Nomenclature

$D_0 + D_u \in R^{n\times n}$	= damping matrix
$F \in R^n$	= control torques/forces
$G_0 + G_u \in R^{n\times n}$	= stiffness matrix
$M_0 + M_u \in R^{n\times n}$	= symmetric, positive definite inertia matrix
$\mathcal{N} \in R^n$	= external disturbance
$p \in R^s$	= uncertain system parameters
$x, \dot{x}, \ddot{x} \in R^n$	= positions/velocities/accelerations

I. Introduction

IN general, control strategy design for a given system is based upon the mathematical model of the system. Obtaining a "good" model for such a system is important for effective control. However, deriving an exact model is almost impossible, and one must use approximate models. Furthermore, even if the underlying physical system could be modeled accurately at one point in time, parameter variations during the operation of the system eventually render the model inaccurate. This is particularly true for large space structures,[1–5] where parameter variations and structural deformation are encountered. It is typical for large space structures to possess high modal density and low inherent damping. For example, the structure of a high-speed flight vehicle is generally designed to have a long fuselage with a small cross-sectional area to reduce aerodynamic drag. Such a structure exhibits considerable flexibility and vibration, especially when the vehicle travels at a high speed or undergoes external disturbances.

To guarantee safe mission in aerospace travel, it is imperative to design an effective control strategy that eliminates or reduces the structural vibration during operation. The past few years have witnessed a large collection of research results in the literature associated with control and stabilization of flexible space structures.[1–3,6–9] Among these efforts, a common practice in addressing the problem of vibration suppression is to use the finite dimensional approach. This method allows the linear system theory to be applied in control design and analysis. Note that in this approach it is generally assumed that the damping matrix is proportional to either the structural mass matrix and/or stiffness matrix. Such a treatment is necessary for the modal analysis approach to permit a decoupling transformation.[3] However, in practice, such an uncoupling transformation is not readily applicable due to the presence of nonlinear effects in real structures. Another approach is to treat a large space structure as partly rigid and partly flexible, so that the complete mathematical model is described by a combination of both ordinary differential equations and a hyperbolic partial differential equation. This is referred to as the infinite dimensional analysis approach.[3] However, to the best of our knowledge, rigorous and feasible results obtained by this approach are very limited due to the complexity of partial differential equations.[1–3]

It is noted that efforts have been made recently in controller design for large space structures by using robust and/or adaptive techniques. Results along this line can be found in Refs. 10–13 (just to mention a few). The common feature of these results is that a precise system model is not required. However, most of these works only address stability, and the overall system performance achieved by these control laws has seldom been discussed.

In this paper, we propose a control scheme for a class of dynamic systems associated with flexible space structures. Both system stability and performance are analyzed. The proposed control scheme consists of three parts: nominal control, feedback control, and auxiliary control. The nominal control component is based on the nominal system parameters and is used to maintain stability of the system operating at nominal conditions. The feedback control is synthesized to enhance system performance. The role of the auxiliary control is to deal with modeling uncertainty and external disturbance. It is

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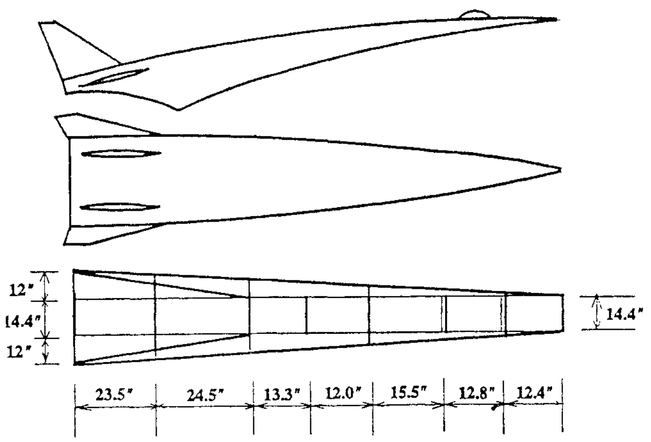

Fig. 1 Generic hypersonic aerospace plane model.

shown that this strategy not only guarantees system stability in the presence of uncertainty/disturbance but also achieves a bounded performance index. Stability is addressed by Lyapunov's direct method. The overall system performance is evaluated by using an index consisting of an integral of a weighted-squared-error. The remainder of this paper is organized as follows. Section II discusses the dynamic equation of the system and formulates the control problem. Since both stabilization and tracking control are of importance in aerospace systems, a unified tracking problem is considered. Section III is devoted to the development of the control strategy, and two control algorithms are introduced to achieve the control objectives. Stability analysis and performance assessment are also addressed. To illustrate the effectiveness of the algorithms, vibration control of a generic hypersonic aerospace flight vehicle, as shown in Fig. 1, is presented in Sec. IV. Finally, the conclusions are stated in Sec. V.

II. Problem Statement

Consider a dynamic system governed by the following nonlinear second-order differential equation,

$$[M_0 + M_u(p, x, t)]\ddot{x} + [D_0 + D_u(p, x, \dot{x}, t)]\dot{x} + [G_0 + G_u(p, t)]x = F + \mathcal{N}(t) \tag{1}$$

In this equation, M_0, D_0, and G_0 are known constant nominal matrices; the unknown function $p\colon R \rightarrow \mathcal{P} \subset R^s$ is Lebesgue measurable with $\mathcal{P}$ compact but unknown; $M_u(.)\colon \mathcal{P} \times R^n \times R \rightarrow R^{n\times n}$ is a Caratheodory[12] inertia matrix with uncertain elements, introduced by fuel consumption and/or structure deformation; $D_u(.)\colon \mathcal{P} \times R^n \times R^n \times R \rightarrow R^{n\times n}$ is a Caratheodory damping matrix with uncertain elements, arising from active/external damping and/or rotary inertia; and $G_u(.)\colon \mathcal{P} \times R \rightarrow R^{n\times n}$ is a Caratheodory matrix with uncertain elements, introduced by material stiffening effects. Note that in this model $\dim(F) = \dim(x)$, which physically implies that the number of actuators is equal to the number of vibrational modes. For the case that $\dim(F) < \dim(x)$, readers are referred to Ref. 20 for details.

Motivation of this type of model stems from the fact that many practical systems can be treated as in Eq. (1), e.g., robotic systems,[18] turbine-machinery,[19] and large space structures.[10] Note that the quantities M_u, D_u, and G_u are added to the model to reflect the effects of uncertain system dynamics. These uncertainties may be related to, for instance, fuel depletion or shifting from one section to another in the structure. The change in the stiffness matrix may also occur due to structural fatigue in the rods or a failure of a member within the structure itself. In general, it is not trivial to derive the precise expressions for these matrices since they may take the form[9]

$$M_u = \int_v \xi[\pi(x), \Omega]\, \mathrm{d}v \tag{2}$$

$$D_u = \frac{\partial}{\partial t}\int_l f(w, x, l, \pi, t)\, \mathrm{d}l \tag{3}$$

when the flexibility of the structure is considered, where $\xi(.)$ is a characteristic function, Ω is the weight density, π is a parametric vibrations, l/v is the length/volume of the structure, and w is a certain weighting function.

At this point, it should be obvious that because of the difficulties in accurately modeling the structural dynamics, a more dedicated control scheme that maintains stability and performance specifications in the presence of large structural uncertainties and variable structural characteristics is highly desired. Hence in this study we assume that only nominal matrices of the system are available. Since both stabilization control and tracking control are of importance for aerospace systems, we will consider the following problems.

Problem 1: Stabilization. Find a control input F such that $x \rightarrow 0$ and $\dot{x} \rightarrow 0$ as $t \rightarrow \infty$. It is also desired that with such a control, a specific performance index is achieved, i.e.,

$$J_1 = \int_0^t \Phi_1(x, \dot{x})\, \mathrm{d}\tau \leq C_1^2 < \infty \tag{4}$$

Note that if x is the displacement of a dynamic structure, the vibration suppression falls into the category of the stabilization problem.

Problem 2: Tracking. Find a control input F such that $x \rightarrow x^d$ and $\dot{x} \rightarrow \dot{x}^d$ as $t \rightarrow \infty$. It is also desired that the given performance index is achieved, i.e.,

$$J_2 = \int_0^t \Phi_2(x, \dot{x}, x^d, \dot{x}^d)\, \mathrm{d}\tau \leq C_2^2 < \infty \tag{5}$$

where x^d and $\dot{x}^d$ are desired paths to be tracked.

In the context of flight vehicle control, this can be applied to the flight-path tracking problem, such as takeoff and re-orientation maneuvers.

Problem 3: Tracking with Stabilization. Suppose that x is partitioned into two parts: $x_1 \in R^{n_1}$ and $x_2 \in R^{n_2}$ $(n_1 + n_2 = n)$. Find F such that x_1 tracks a given trajectory x_1^d, whereas x_2 converges to an equilibrium. The performance index to be achieved is

$$J_3 = \int_0^t \Phi_3\left(x, \dot{x}, x_1^d, \dot{x}_1^d\right) \mathrm{d}\tau \leq C_3^2 < \infty \tag{6}$$

Obviously, under mild conditions these problems can be presented in the following unified fashion.

Find F such that as $t \rightarrow \infty$, $e \rightarrow 0$, and $\dot{e} \rightarrow 0$ (e and $\dot{e}$ denote tracking error and velocity, respectively), whereas a bounded performance index

$$J = \int_0^t \Phi(e, \dot{e})\, \mathrm{d}\tau \leq C^2 < \infty \tag{7}$$

is ensured.

As a matter of fact, choosing $x^d = 0$ (hence $\dot{x}^d = 0$, $\ddot{x}^d = 0$) leads one to problem 1, whereas problem 3 is the case where x_1^d, $\dot{x}_1^d$, and $\ddot{x}_1^d$ are given, and x_2^d is set identically zero. We will concentrate on this tracking problem in the material that follows. For later technical development, the following notations are adopted.

Let $A(x)$ be a symmetric positive definite matrix; $\lambda_M(A)$ and $\lambda_m(A)$ denote its maximum and minimum eigenvalues for any $x \in R^n$. The norm of an $n \times 1$ vector, x, is defined as

$$\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$$

and the norm of a matrix A is defined as the corresponding induced norm

$$\|A\| = \left[\lambda_M(A^T A)\right]^{\frac{1}{2}}$$

In addition, we need the following lemmas for analyzing the stability of the controller.

Lemma 2.1. Let V be a C^1 time function defined on $[0, T)$ $(0 < T \leq +\infty)$, satisfying

$$\dot{V} \leq -cV + \gamma_1(t)$$

where c is a strictly positive constant and γ is a positive time function belonging to $L_2(0, T)$, i.e.,

$$\int_0^T \gamma_0^2 \, d\tau \le s_1 < \infty$$

Under these assumptions, $V(t)$ is upper bounded on $[0, T)$ and is given as

$$V(t) \le V(0) + \sqrt{\frac{2}{c}}\sqrt{s_1}, \qquad \forall t \in [0, T)$$

Moreover,

$$\limsup_{t \to \infty} V(t) \le 0$$

Proof. See Ref. 16. □

Lemma 2.2. Let V_1: $R^n \to R$ and $V_2(\hat{\rho})$: $R^m \to R$ be nonnegative and C^1 time functions defined on $[0, T)(0 < T \le +\infty)$, satisfying

$$\lambda_1 \|x\|^2 \le V_1(x) \le \lambda_2 \|x\|^2 \qquad \forall \lambda_2 \ge \lambda_1 > 0$$

$$\lambda_3 \|\hat{\rho}\|^2 \le V_2(\hat{\rho}) \le \lambda_4 \|\hat{\rho}\|^2 \qquad \forall \lambda_4 \ge \lambda_3 > 0$$

$$\dot{V}_1 + \dot{V}_2 \le -\delta_1 V_1 - \delta_2(t) V_2 + \gamma_0(t) \tag{8}$$

where δ_1 is a strictly positive constant, $\delta_2(t) \ge 0$, and $\gamma_0(t)$ is a time function belonging to $L_1(0, T)$, i.e.,

$$\int_0^T \gamma_0 \, d\tau \le s_0 < \infty$$

then $x \in L_2 \cap L_\infty$ and $\hat{\rho} \in L_\infty$. Moreover, if x is uniformly continuous function of t, $\|x\| \to 0$ as $t \to \infty$.

Proof. Integration of both sides of Eq. (8) gives

$$V_1(t) - V_1(0) + V_2(t) - V_2(0)$$
$$\le -\delta_1 \int_0^t V_1 \, d\tau - \int_0^t \delta_2(\tau) V_2 \, d\tau + s_0 \tag{9}$$

from which we have

$$V_1 \le V_1(0) + V_2(0) - V_2 - \delta_1 \int_0^t V_1 \, d\tau - \int_0^t \delta_2(\tau) V_2 \, d\tau + s_0$$
$$\le V_1(0) + V_2(0) + s_0 \tag{10}$$

where the facts that $V_1(.) \ge 0$, $V_2(.) \ge 0$, and $\int_0^t \delta_2(\tau) V_2 \, d\tau \ge 0$ were used. Noting that $\lambda_1 \|x\|^2 \le V_1$, it is then obtained from Eq. (10) that

$$\|x\|^2 \le \frac{1}{\lambda_1}[V_1(0) + V_2(0) + s_0] < \infty$$

implying that $x \in L_\infty$. Similarly, we can show that

$$V_2 \le V_1(0) + V_2(0) + s_0$$

and then $\|\hat{\rho}\|^2 \le (1/\lambda_3)[V_1(0) + V_2(0) + s_0] < \infty$, i.e., $\hat{\rho} \in L_\infty$. Furthermore, from Eq. (9) it can be derived that

$$\int_0^t V_1 \, d\tau \le \frac{1}{\delta_1}[V_1(0) + V_2(0) + s_0]$$

which, upon using the relation $\lambda_1 \|x\|^2 \le V_1$, reduces to

$$\int_0^t \|x\|^2 \, d\tau \le \frac{1}{\lambda_1 \delta_1}[V_1(0) + V_2(0) + s_0] < \infty$$

indicating that $x \in L_2$. Since x is uniformly continuous, $x \in L_2 \cap L_\infty$ implies that $\|x\| \to 0$ as $t \to \infty$ by Barbalat lemma.[14] □

III. Development of Control Strategy

To propose a feasible control strategy, the following assumptions concerning the dynamic system (1) are imposed.

Assumption 1. There exist some constants, $0 \le \rho_i < \infty$ $(i = 0, 1, 2)$ such that

$$\sup_{t \in [0,\infty)} \|M_u\| \le \rho_0$$

$$\sup_{t \in [0,\infty)} \|\dot{M}_u\| \le \rho_1 \psi_1(\|x\|, \|\dot{x}\|)$$

$$\sup_{t \in [0,\infty)} \|D_u\| \le \rho_2 \psi_2(\|x\|, \|\dot{x}\|)$$

$$\sup_{t \in [0,\infty)} \|G_u\| \le \rho_3 \psi_3(\|x\|)$$

where $\psi_i(.)$ $(i = 1, 2, 3)$ are known positive scalar functions.

Assumption 2. The states of the system are available for feedback control.

Assumption 3. The external disturbance is upper bounded in the sense that there exists some constant $\rho_4 < \infty$ such that

$$\sup_{t \in [0,\infty)} \|\mathcal{N}\| \le \rho_4$$

Remark. Assumption 1 is automatically satisfied for a broader class of nonlinear systems (e.g., systems that obey Lagrange-Euler formulation). Note that Assumption 2 is restrictive for general dynamic systems, and relaxing this assumption is an interesting area for further work. Assumption 3 is necessary because unbounded disturbances would render the system extremely difficult to control.

Two control algorithms are developed in this section. In each algorithm the following control functions are used,

$$e = x - x^d$$

$$W = \dot{e} + \beta e$$

$$\mathcal{X} = \ddot{x}^d - (\alpha + \beta)\dot{e} - \alpha\beta e$$

where $\alpha > 0$ and $\beta > 0$. Note that e and $\dot{e}$ are the tracking errors and W is a function associated with the weighted tracking errors, while $\mathcal{X}$ is an auxiliary signal that is available as long as x and $\dot{x}$ are measurable.

To evaluate the system behavior, we consider the performance index

$$J(e, \dot{e}) = \int_0^t (a\dot{e}^T \dot{e} + b e^T e + c e^T \dot{e}) \, d\tau$$

where $a > 0$, $b > 0$, and $c > 0$ are weighting factors. The motivation of choosing the preceding performance index is that this index represents the "integral of weighted-squared-error (IWSE)" and thus reflects the overall control qualities in terms of transient and steady-state performance. This index is commonly used in practice.[15,16] It is interesting to note that if the weighting factors are set as

$$a = 1, \qquad b = \beta^2, \qquad c = 2\beta$$

the performance index takes the form

$$J(e, \dot{e}) = \int_0^t \|W\|^2 \, d\tau$$

For simplicity, the following performance assessment is based on this index. Similar analysis can be addressed for the general performance index.

The proposed control scheme consists of the following three parts:

$$F = F_n(x, \dot{x}, x^d, \dot{x}^d, \ddot{x}^d) + F_f(x, \dot{x}, x^d, \dot{x}^d) + F_a(x, \dot{x}, x^d, \dot{x}^d) \tag{11}$$

where F_n denotes the nominal control, F_f represents the feedback control, and F_a is the auxiliary control.

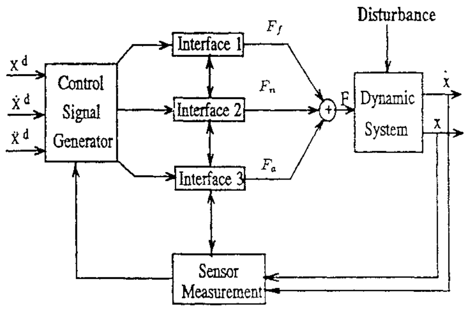

Fig. 2 Adaptive robust control system.

The block diagram of the control strategy is depicted in Fig. 2. The design task is to derive the control signals F_n, F_f, and F_a to achieve the objectives imposed previously. This is detailed in what follows.

A. Robust Control Algorithm

The first control algorithm is constructed as follows,

$$F_n = M_0\mathcal{X} + D_0\dot{x} + G_0 x \tag{12}$$

$$F_f = -KW \tag{13}$$

$$F_a = -\frac{W\eta^2}{\|W\|\eta + \nu(t)} \tag{14}$$

where $K = K^T > 0$, chosen by the designer, and η is defined by

$$\eta = \rho_0\|\mathcal{X}\| + \rho_1\psi_1(\|x\|, \|\dot{x}\|)\|W\| + \rho_2\psi_2(\|x\|, \|\dot{x}\|)\|\dot{x}\| + \rho_3\psi(\|x\|)\|x\| + \rho_4 \tag{15}$$

and $\nu(t)$ is a design function belonging to $L_1 \cap L_2[0, \infty)$, i.e.,

$$0 < \int_0^t \nu^2(\tau)\,d\tau \le d_2 < \infty \qquad \forall t \in [0, \infty) \tag{16}$$

$$0 < \int_0^t \nu(\tau)\,d\tau \le d_1 < \infty \qquad \forall t \in [0, \infty) \tag{17}$$

Theorem 3.1. Consider the system (1) subject to Assumptions 1–3. If the control algorithms as specified in Eqs. (11–17) are implemented, then the control objectives in terms of convergence of tracking errors and bounded performance index J are ensured.

Proof. Application of the control law as specified in Eq. (11) leads to the closed-loop system

$$[M_0 + M_u](\dot{W} + \alpha W) = -KW + F_a - L_1 \tag{18}$$

where

$$L_1 = M_u\mathcal{X} + D_u\dot{x} + G_u x - \mathcal{N} \tag{19}$$

Using the fact that $M_0 + M_u$ is symmetric and positive definite, the following function

$$V_1(W) = \tfrac{1}{2}W^T[M_0 + M_u]W \tag{20}$$

can be constructed as a candidate for the Lyapunov function. Differentiating this function along the solution of Eq. (18) gives

$$\begin{aligned}\dot{V}_1 &= \tfrac{1}{2}W^T\dot{M}_u W + W^T[M_0 + M_u]\dot{W}\\ &= \tfrac{1}{2}W^T\dot{M}_u W + W^T\{-KW + F_a - L_1 - \alpha(M_0 + M_u)W\}\\ &= -2\alpha V_1(W) - W^T K W + W^T[F_a - L_1 + L_2]\end{aligned} \tag{21}$$

where

$$L_2 = 0.5\dot{M}_u W$$

With Assumption 1, it is straightforward to show that

$$\|L_1\| \le \rho_0\|\mathcal{X}\| + \rho_2\psi_2(\|x\|, \|\dot{x}\|)\|\dot{x}\| + \rho_3\psi_3(\|x\|)\|x\| + \rho_4 \stackrel{\Delta}{=} \eta_1$$

$$\|L_2\| \le \rho_1\psi_1(\|x\|, \|\dot{x}\|)\|W\| \stackrel{\Delta}{=} \eta_2$$

Obviously,

$$\|L_2 - L_1\| \le \eta_1 + \eta_2 = \eta \tag{22}$$

Therefore

$$\begin{aligned}\dot{V}_1 &\le -2\alpha V_1 - W^T K W + \|W\|\|L_2 - L_1\| + W^T F_a\\ &\le -(2\alpha + \delta)V_1 + \|W\|\eta + W^T F_a\end{aligned}$$

where $\delta = \{[\lambda_m(K)]/[\lambda_M(M_0 + M_u)]\} > 0$. Recall that F_a is given by Eq. (14); then we have

$$\begin{aligned}\|W\|\eta + W^T F_a &= \|W\|\eta + W^T\left\{-\frac{W\eta^2}{\|W\|\eta + \nu(t)}\right\}\\ &= \frac{\|W\|\eta\nu}{\|W\|\eta + \nu} \le \nu\end{aligned}$$

which reduces $\dot{V}_1$ to

$$\dot{V}_1 \le -(2\alpha + \delta)V_1 + \nu(t) \tag{23}$$

Note that $2\alpha + \delta$ is a strictly positive constant and $\nu \in L_2$, by Lemma 2.1 it is concluded that

$$V_1(W) \to 0 \text{ as } t \to \infty$$

Consequently, $W \to 0$ as $t \to \infty$. Since $W = \dot{e} + \beta e$ and $\beta > 0$, it can be readily concluded that both $\dot{e}$ and e converge to zero as time increases. To complete the proof, we still need to show the boundedness of the performance index $J = \int_0^t \|W\|^2\,d\tau$. From Eq. (18),

$$\begin{aligned}J_K &\stackrel{\Delta}{=} \int_0^t W^T K W\,d\tau\\ &= -\alpha\int_0^t W^T(M_0 + M_u)W\,d\tau\\ &\quad - \int_0^t W^T(M_0 + M_u)\dot{W}\,d\tau + \int_0^t W^T(F_a - L_1)\,d\tau\\ &\le -\int_0^t W^T(M_0 + M_u)\dot{W}\,d\tau + \int_0^t W^T(F_a - L_1)\,d\tau\end{aligned}$$

Carrying out integration by parts and noting that $M_0 + M_u$ is symmetric and positive definite, it is not difficult to show that

$$-\int_0^t W^T(M_0 + M_u)\dot{W}\,d\tau \le \tfrac{1}{2}W^T(M_0 + M_u)W|_{t=0} + \frac{1}{2}\int_0^t W^T\dot{M}_u W\,d\tau$$

Using this relation,

$$\begin{aligned}J_K &\le C_0^2 + \int_0^t W^T(F_a - L_1 + L_2)\,d\tau\\ &\le C_0^2 + \int_0^t W^T F_a + \int_0^t \|W\|\eta\,d\tau\end{aligned}$$

where

$$C_0^2 = \tfrac{1}{2}W^T(M_0 + M_u)W|_{t=0}$$

Since F_a is defined as in Eq. (14) in which $\nu(t)$ satisfies Eq. (17), it is readily verified that

$$J_K \le C_0^2 + \int_0^t \frac{\|W\|\eta\nu}{\|W\|\eta + \nu}\,d\tau \le C_0^2 + \int_0^t \nu(\tau)\,d\tau \le C_0^2 + d_1$$

Recall that $\lambda_{\min}(K)\|W\|^2 \le W^T K W$; we then have

$$J(e,\dot e) = \int_0^t \|W\|^2\,d\tau \le \frac{1}{\lambda_m(K)}\left(C_0^2 + d_1\right) < \infty \tag{24}$$

□

Remarks. 1) It is worth mentioning that in our development the second-order differential equation as represented by Eq. (1) is directly considered without transforming it into a first-order state-space form. Such a treatment allows us to make use of the symmetric and positive definite property of the matrix $M_0 + M_u$, which plays an important role in stability and performance analysis.

2) As can be observed from the proof of Theorem 3.1, the nominal control and the PD feedback control ensure system stability under a nominal operation condition. However, whenever there exist external disturbances or modeling uncertainties, it is necessary to introduce an auxiliary control to effectively subdue the effects arising from system uncertainties.

3) In order for the proposed algorithm to guarantee tracking stability and boundedness of performance index, $\nu(t)$ has to be L_2 and L_1. It is interesting to note that there are many possible choices for such a ν, e.g., a) $\nu(t) = 1/(1+t^{\alpha_0})$, b) $\nu(t) = 1/(1+t)^{\alpha_0}$, or c) $\nu(t) = e^{-\upsilon_1 t}$, where $\alpha_0 > 1$ and $\upsilon_1 > 0$.

4) It is seen that the overall control performance with this strategy depends on the initial system conditions C_0^2 and the control gain K. Clearly, in this strategy, increasing $\|K\|$ will eventually enhance the system performance.

B. Adaptive Robust Control Algorithm

Note that to implement the proposed robust control algorithm, ρ_i $(i = 0, 1, \ldots, 4)$ should be available. However, in some cases, the procedure to find ρ_i may be very tedious. This is particularly true when there involves a large number of vibrational modes in flexible structures. In the following a strategy that does not require the information of ρ_i is proposed. In this scheme, an adaptive algorithm is constructed to estimate these parameters.

First we define

$$\rho = [\rho_4\ \rho_3\ \rho_2\ \rho_1\ \rho_0]^T$$

$$\psi(x,\dot x) = [1\ \psi_3(.)\|x\|\ \psi_2(.)\|\dot x\|\ \psi_1(.)\|W\|\ \|\mathcal{X}\|]$$

$$r(\psi) = 1 + \psi_3(.)\|x\| + \psi_2(.)\|\dot x\| + \psi_1(.)\|W\| + \|\mathcal{X}\|$$

Theorem 3.2. Consider the dynamic system (1) with Assumptions 1–3. Let the control input be defined as in Eq. (11) in which F_n and F_f are given by Eqs. (12) and (13), and F_a is specified as follows,

$$F_a = -\frac{W\hat\eta}{\|W\| + \mu(t)} \tag{25}$$

with

$$\hat\eta = \hat\rho^T\psi \tag{26}$$

$$\mu(t) = \frac{\nu(t)}{1 + r(\|\psi\|)} \tag{27}$$

$$\dot{\hat\rho} = -\sigma\hat\rho + g\frac{\|W\|^2}{\|W\| + \mu}\psi \tag{28}$$

where $g > 0$ is a constant. If ν is chosen as in Eqs. (16) and (17) and σ is chosen to satisfy

$$\sigma > 0 \text{ and } \int_0^t \sigma\,d\tau \le c_1 < \infty \tag{29}$$

then asymptotically tracking is achieved and the bounded performance index J is guaranteed.

Proof. Consider the following Lyapunov candidate,

$$V_2(W, \hat\rho - \rho) = V_1(W) + \frac{1}{2g}(\hat\rho - \rho)^T(\hat\rho - \rho) \tag{30}$$

where V_1 is defined as before. Taking a derivative of Eq. (30) with respect to time along the vector field of Eq. (18) gives

$$\dot V_2 = \dot V_1 + \frac{1}{g}(\hat\rho - \rho)^T\dot{\hat\rho} = -(2\alpha+\delta)V_1 + W^T(F_a + L_2 - L_1) + \frac{1}{g}(\hat\rho - \rho)^T\dot{\hat\rho} \tag{31}$$

Noting that the control signal F_a is generated by Eq. (25) and using the fact that $\|L_2 - L_1\| \le \eta = \rho^T\psi$ yields

$$\begin{aligned}\dot V_2 &\le -(2\alpha+\delta)V_1 + \|W\|\rho^T\psi + W^T\left(-\frac{W\hat\rho^T\psi}{\|W\|+\mu}\right) + \frac{1}{g}(\hat\rho-\rho)^T\dot{\hat\rho}\\ &= -(2\alpha+\delta)V_1 - (\hat\rho-\rho)^T\frac{\|W\|^2\psi}{\|W\|+\mu} + \frac{\|W\|}{\|W\|+\mu}\mu\rho^T\psi + \frac{1}{g}(\hat\rho-\rho)^T\dot{\hat\rho}\\ &\le -(2\alpha+\delta)V_1 + \frac{1}{g}(\hat\rho-\rho)^T\left[\dot{\hat\rho} - g\frac{\|W\|^2\psi}{\|W\|+\mu}\right] + \mu\rho^T\psi\end{aligned}$$

where the fact that $\|W\|/(\|W\|+\mu) \le 1$ is used. With $\hat\rho$ being updated by Eq. (28), $\dot V_2$ reduces to

$$\dot V_2 \le -(2\alpha+\delta)V_1 - \frac{\sigma}{g}(\hat\rho-\rho)^T\hat\rho + \mu\rho^T\psi \tag{32}$$

Referring to the definition of $r(.)$ as given before, it can be verified that

$$\rho^T\psi \le \rho_M r$$

where $\rho_M = \max_i(\rho_i)$, which leads to

$$\mu\rho^T\psi \le \frac{\nu}{1+r}\rho_M r = \nu\rho_M \tag{33}$$

Also note that

$$-\frac{\sigma}{g}(\hat\rho-\rho)^T\hat\rho = -\frac{\sigma}{2g}(\hat\rho-\rho)^T(\hat\rho-\rho) - \frac{\sigma}{2g}\hat\rho^T\hat\rho + \sigma\frac{\rho^T\rho}{2g} \le -\frac{\sigma}{2g}(\hat\rho-\rho)^T(\hat\rho-\rho) + \sigma\frac{\rho^T\rho}{2g} \tag{34}$$

Combining Eq. (32) with Eqs. (33) and (34) gives

$$\dot V_2 \le -(2\alpha+\delta)V_1 - \frac{\sigma}{2g}(\hat\rho-\rho)^T(\hat\rho-\rho) + \sigma\frac{\rho^T\rho}{2g} + \nu\rho_M = -\delta_1 V_1 - \delta_2 V_2 + \gamma_0 \tag{35}$$

where

$$\delta_1 = 2\alpha + \delta > 0 \quad \forall t$$

$$\delta_2 = \sigma \ge 0 \quad \forall t$$

$$\gamma_0 = \sigma\frac{\rho^T\rho}{2g} + \nu\rho_M$$

With the choices of ν and σ as before [i.e., Eqs. (16), (17), and (29)], it is seen that

$$\begin{aligned}\int_0^t \gamma_0 \,\mathrm{d}\tau &= \int_0^t \left(\rho_M \nu + \frac{\sigma\rho^T\rho}{2g}\right)\mathrm{d}\tau \\ &= \rho_M \int_0^t \nu \,\mathrm{d}\tau + \frac{(\rho^T\rho)}{2g}\int_0^t \sigma \,\mathrm{d}\tau \\ &\le \rho_M d_1 + \frac{(\rho^T\rho)}{2g}c_1 \\ &\triangleq s_0 < \infty \end{aligned}\tag{36}$$

Then by Lemma 2.2, $\hat{\rho} - \rho \in L_\infty$, i.e., $\hat{\rho}$ is bounded, and $W \in L_2 \cap L_\infty$. Furthermore, from Eq. (18) we see that

$$(M_0 + M_u)[\dot{W} + \alpha W] = \mathcal{U}$$

where $\mathcal{U} \in L_\infty$. Notice that $M_0 + M_u$ is invertible and bounded; we then have $\dot{W} \in L_\infty$, i.e., W is uniformly continuous. Thus by Lemma 2.2, $\|W\| \to 0$ as $t \to \infty$, which leads to asymptotic tracking stability. To show the boundedness of the performance index, $J = \int_0^t \|W\|^2 \,\mathrm{d}\tau$, we note that from the foregoing development, $\dot{V}_2$ can be written as

$$\begin{aligned}\dot{V}_2 &\le -\alpha W^T(M_0 + M_u)W - W^T K W + \gamma_0 \\ &\le -[\alpha\lambda_m(M_0 + M_u) + \lambda_m(K)]\|W\|^2 + \gamma_0\end{aligned}\tag{37}$$

Integration of both sides of Eq. (37) yields

$$\begin{aligned}&[\alpha\lambda_m(M_0 + M_u) + \lambda_m(K)]\int_0^t \|W\|^2 \,\mathrm{d}\tau \\ &\le V_2(0) + \int_0^t \gamma \,\mathrm{d}\tau \\ &= V_2(0) + \frac{\rho^T\rho}{2g}\int_0^t \sigma \,\mathrm{d}\tau + \rho_M \int_0^t \nu \,\mathrm{d}\tau \\ &\le V_2(0) + \frac{\rho^T\rho}{2g}c_1 + \rho_M d_1 < \infty\end{aligned}$$

Therefore

$$\begin{aligned}J(e, \dot{e}) &= \int_0^t \|W\|^2 \,\mathrm{d}\tau \\ &\le \frac{V_2(0) + (\rho^T\rho/2g)c_1 + \rho_M d_1}{\alpha\lambda_m(M_0 + M_u) + \lambda_m(K)} < \infty\end{aligned}\tag{38}$$

which completes the proof. □

Remarks. 1) The control algorithm has its origin in Refs. 17 and 18 and thus shares similar features as described therein. However, note that our strategy exhibits additional features. That is, with this strategy, asymptotic stability, instead of ultimately bounded stability, is achieved. Our strategy also guarantees bounded performance index.

2) If $\nu(t)$ is chosen as a small constant, bounded tracking error is obtained. The performance index in this case is no longer bounded. Only the average performance index as defined by $\lim_{t\to\infty}[J/t - t_0]$ is bounded.

3) The advantage of the adaptive robust control algorithm over the robust one is that the design procedure is much simpler in that no pre-estimation of the various parameters ρ_i is needed. The overall strategy is therefore model independent, robust, and adaptive.

4) From the evaluation of the system performance [refer to Eq. (38)] we see that by increasing $\|K\|$ or g, better performance can be achieved. Physically, increasing $\|K\|$ is equivalent to increasing the control energy and increasing g implies that the adaptation speed is increased, which of course leads to a better control quality.

5) It is interesting to note that the foregoing performance evaluation also allows us to address the effect of the initial estimation of $\hat{\rho}_i(0)$ on system performance. Traditionally, it is suggested that the initial estimate may be chosen arbitrarily (zero in general). This is because the stability is global, and thus the initial estimate does not affect tracking stability. However, from the definition of V_2 we see that

$$V_2(0) = C_0^2 + \frac{1}{2g}\sum_{i=0}^{4}[\hat{\rho}_i(0) - \rho_i]$$

which implies from Eq. (38) that the initial estimate affects the overall control quality in the sense that a "better" initial estimate, $\hat{\rho}_i(0)$, results in a tighter bound J. Simply choosing $\hat{\rho}_i(0) = 0$, as typically suggested in the literature, is among the "worst" choices. Choosing the nominal value of ρ_i as the initial estimate results in a smaller J, implying a better performance.

IV. Design Example

The control algorithms presented earlier are verified in this section through a numerical example. The model considered here is a lumped mass model of a generic hypersonic aerospace flight vehicle. It is noted that the real structure has infinite degrees of freedom. The complete dynamics of such a system, taking into account flexibility of the fuselage, aeroelastic effects, and the internal dynamics of the engine/actuator control surfaces, are complicated and unmanageable for control design. A useful first approximation to this type of problem is to start with a relatively simple model. In our study, a simplified structure, as illustrated in Fig. 1, is used for simulation. In this model, the real structure is conceptualized and replaced by an ideal model consisting of a number of lumped masses. These are connected to each other through elastic massless elements that, to a certain extent, retain the nominal behavior of the original structure. Hence, the mass matrix, damping matrix, and stiffness matrix obtained by the lumped mass method can be treated as nominal matrices of the system. The structure is constructed from a material with a modulus of elasticity $E = 10^7$ psi and a weight density of $\Omega = 0.1$ lbf/in.[3] It is important to note that nonstructural mass, i.e., the mass of the actuators, plays a significant role in system dynamics. The nonstructural mass is large as compared with the structural

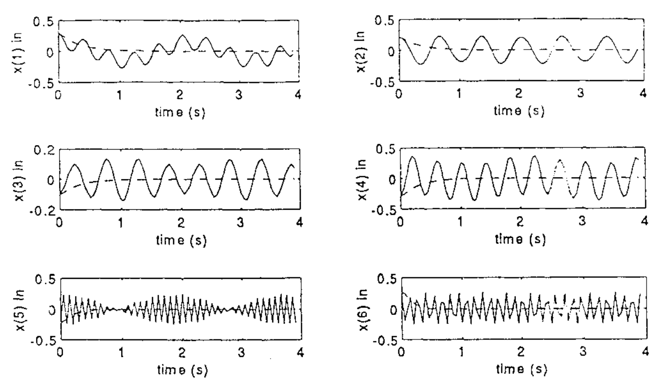

Fig. 3 Vibration modes (· · ·, with control; —, without control).

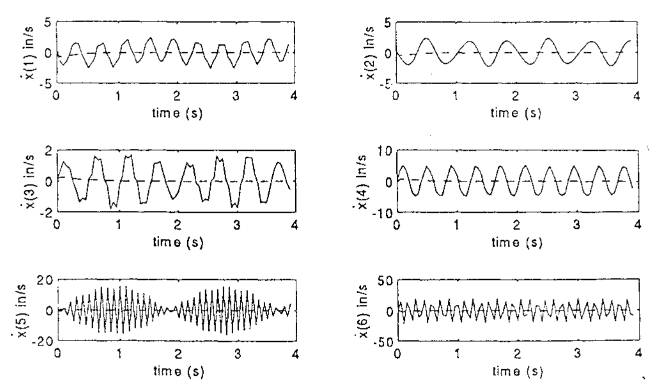

Fig. 4 Vibration rates (· · ·, with control; —, without control).

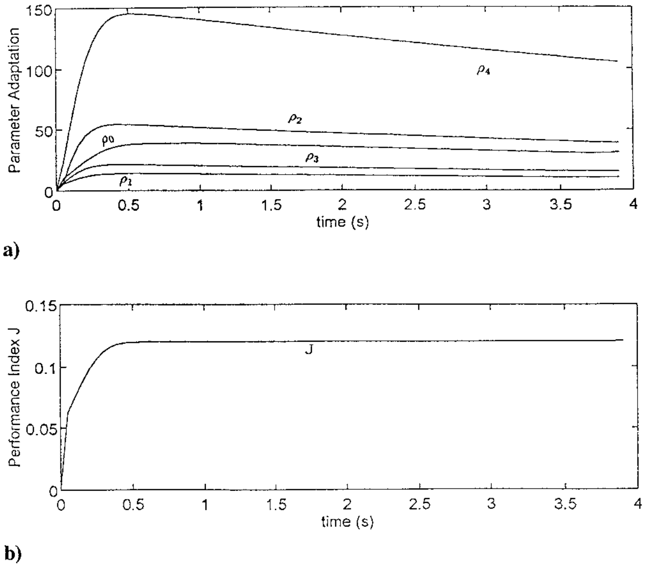

Fig. 5 Parameter adaptation and the performance index.

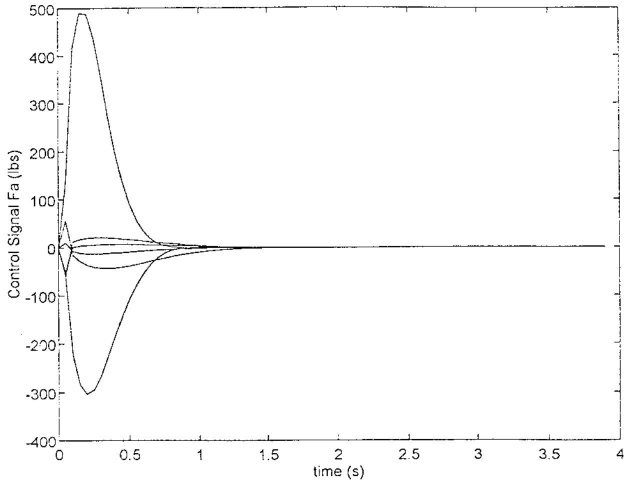

Fig. 6 Auxiliary control input.

mass. In the study, the mass of each actuator is assumed to be 8.4 (lbf-s^2/in.). The nominal inertia matrix is given as

$$M_0 = \mathrm{diag}\{8.4, 8.4, 8.4, 8.4, 8.4, 8.4\}$$

Note that the inherent structural damping for a hypersonic aerospace vehicle is very low. Thus to reflect this fact the damping matrix is assumed to be negligible, i.e.,

$$D_0 = 0_{6\times 6}$$

The nominal stiffness matrix is given as

$$G_0 =$$

$$\begin{bmatrix} 360.00 & -36.00 & 0.00 & -689 & -689 & 0.00 \\ -36.00 & 730.00 & -36.00 & 69.00 & 0.00 & -69.00 \\ 0.00 & -36.00 & 1270.00 & 0.00 & 690.00 & -206.00 \\ -689.00 & 69.00 & 0.00 & 1747.00 & 873.00 & 0.00 \\ -689.00 & 0.00 & 690.00 & 873.00 & 34951 & 873.00 \\ 0.00 & -69.00 & -206.00 & 0.00 & 873.00 & 52426 \end{bmatrix}$$

It is desired that the structural vibration be attenuated to an acceptable level. To that end, the adaptive robust control algorithm is used in the simulation. Since this is a stabilization problem, x^d, $\dot{x}^d$, and $\ddot{x}^d$ are chosen to be zero. The controller parameters are chosen as $\beta = 3.2$ and $\alpha = 1.5$. The control gain K is $K = \mathrm{diag}\{200, 200, 200, 200, 200, 200\}$, and the adaptive gain is set to be $g = 200$. Note that in this study, a lumped uncertainty/disturbance of the form

$$\mathcal{N}(t, x, \dot{x}) = \sin(t)C + A_2 x + A_3 \dot{x} + \cos(t)(x + \dot{x})$$

is artificially added to the nominal model, where $C \in R^6$ is a vector varying within [0, 12], and A_2 and A_3 are scalars within [0, 8] and [0, 15], respectively. To test the effectiveness of the strategy, perturbation of the form

$$p = p_0[1 + \delta_p \sin(t)]$$

on each element of G_0, D_0, and M_0 is introduced in the model, where p_0 represents the nominal value and $\delta_p = 0.5$ is used in simulation. The initial conditions of the displacement and the velocity are $x(0) = [0.29\ 0.21\ -0.1\ -0.28\ -0.21\ 0.27]^T$ (in.) and $\dot{x}(0) = [0.1\ 0.14\ 0.1\ -0.1\ 0.11\ 0.08]^T$ (in./s), respectively. For comparison, the results with and without the proposed control are depicted in the same plot, as shown in Figs. 3 and 4. Figure 3 shows the suppression of the six vibration modes (modal amplitude). Figure 4 is the attenuation of modal velocities. Figure 5a shows the adaptation of the five generalized scalar parameters $\hat{\rho}_i$. The overall control performance index J is presented in Fig. 5b. The auxiliary control signals F_a are depicted in Fig. 6. It is seen that each element of F_a is continuous, which leads to smooth overall control action. The effectiveness of the strategy in terms of robustness and adaptation is obvious from these results. The system quickly reaches a steady state, and uncertainties and disturbances are effectively rejected.

V. Conclusions

Model-based control cannot guarantee satisfactory performance for systems having parameter variations and external disturbances. In this paper, two new control algorithms are developed with particular attention to subduing the effects of modeling uncertainty. The proposed strategy can be used for tracking and stabilization purposes. It is shown that a bound on the performance index can also be guaranteed with the scheme. The results are verified via application to vibration suppression of a generic hypersonic aerospace vehicle modeled by the lumped mass approach. It should be mentioned that due to the nonlinear nature of the system, the achievement of the specified performance with the proposed approach requires both position and velocity measurements. Extension of the results to output feedback case represents an interesting area for further research.

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